Higher-Order Terms in Bivariate Returns to International Stock Market Indices

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This article documents the stochastic properties of bivariate returns to international stock market indices. In particular, the article searches for the best fit among a class of higher-order VARMA(u,v)-EGARCH(p,q) models with normal errors and a constant conditional correlation using MSCI domestic and world-ex-domestic index pairs for the Emu, Japan, the United Kingdom, and the United States. Although a first-order VAR or VMA specification is sufficient to accommodate the conditional means, second-order EGARCH terms are necessary in two of the four bivariate series (JEL: G15 G11 C15 C34).

Keywords: higher-order, bivariate, international diversification, EGARCH, VARMA.

I. Introduction

This article examines the stochastic properties of bivariate daily returns to the MSCI domestic and world-ex-domestic stock market index pairs for the Emu, Japan, the United Kingdom, and the United States. In particular, the article examines whether higher-order terms are necessary in these series by searching for the best fit among the class of bivariate VARMA(u,v)-EGARCH(p,q) models with a constant conditional correlation and normally distributed errors using conditional mean and volatility terms at lags of up to three days. First-order terms are usually sufficient to capture the conditional mean and volatility of univariate price series (Engle [1993]). First-order models have a more straightforward economic interpretation than higher-order models, and are easier to construct (He, Teräsvirta and Malmsten [2002]) and econometrically

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more tractable (He and Teräsvirta [1999]). The suitability of first-order models for bivariate returns to international indices has not been investigated. Higher-order terms could arise for many reasons, including technical factors such as nonsynchronous measurement of returns (Lo and MacKinlay [1990]) or behavioral factors (Hirshleifer [2001]) such as market contagion (Bae, Karolyi and Stulz [2003]).

Higher-order conditional volatility terms are significant in half of these bivariate series. This is about the same proportion as in the univariate series. Although an EGARCH(1,1) model provides a relatively good fit for bivariate U.K. and U.S. returns, second-order EGARCH terms are useful in the Emu and Japan series. The additional terms have significant coefficients and yield improved residual behaviors and significant robust Wald statistics relative to the EGARCH(1,1) model. The conditional means of these bivariate series can be modeled equally well with first-order vector autoregressive (VAR) or moving average (VMA) terms.

II. Data

In order to take the perspective of a domestic investor considering the diversification benefits of international assets, Morgan Stanley Capital International (MSCI) value-weighted domestic and world-ex-domestic (world return excluding domestic return) stock market indices are employed for the Emu, Japan, the United Kingdom, and the United States. This contrasts with most studies of international returns, which study correlations between national markets. The bivariate distribution of domestic/world-ex-domestic returns is important because it determines the diversification gains to domestic investors from international investments.

The model is estimated using continuously compounded local currency daily returns to MSCI official price indices for the domestic Emu, Japanese, U.K. and U.S. markets and their corresponding world-ex-domestic indices over the period 02/01/1996 through 12/31/2002. These four domestic markets account for about 90 percent of total MSCI stock market capitalization. Local currency returns are used to represent returns earned by domestic investors that are fully hedged against currency risk. As a practical matter, the stochastic properties of local currency and U.S. dollar returns are quite similar.

Some days in the sample period, such as national holidays, have a zero (missing) return for a domestic index and a non-zero return for the corresponding world-ex-domestic index. To preserve the continuity of the bivariate series, non-trading days in each domestic market are aggregated onto the next trading day in that market. Returns to the world-ex-domestic index over these periods are similarly aggregated into a single return so that the world-ex-domestic sample aligns with the domestic index. This convention preserves the perspective of a domestic investor, for whom non-trading days in the domestic market have volatilities that are only a small fraction of the volatilities on trading days (French and Roll [1986]).

Descriptive statistics for each series appear in table 1. Significant skewness and kurtosis indicate that these observed daily returns are not normally distributed. Seven out of eight skewness measures are negative because of a few large negative returns, and five are significant at 5 percent. All eight univariate indices are leptokurtic and significant at 1 percent. These nonnormalities guide the choice of an EGARCH specification to accommodate volatility asymmetry, as well as a robust quasi-maximum likelihood estimation technique in Section III.

Significant first-order autocorrelation is present at one percent in each of the world-ex-domestic indices and at five percent in the Emu index, presumably because the national markets comprising these indices close at different times throughout the day. Six of the eight univariate indices exhibit at least one significant second-order or third-order partial autocorrelation, indicating that price-adjustment delays last longer than one day in these data.

The serial cross correlations reflect the closing times of the various domestic and world-ex-domestic markets. Japan is the first market to open and the United States is the last to close during each calendar day. Thus, observed returns in Japan should be related to the previous day's world-ex-Japan returns and observed U.S. returns should be related to the next day's world-ex-U.S. returns. In table 1, first-order cross correlation is indeed significant at 1 percent when the Emu (0.2062), Japanese (0.3103), or U.K. (0.1821) index lags the corresponding world-ex-domestic index. The first-order cross correlation between the Japanese index and the corresponding world-ex-Japan index (0.3103) actually exceeds the contemporaneous correlation (0.1743). First-order serial cross correlation is only significant for the U.S. index when it is paired with the next day's world-ex-U.S. index (0.4006). Higher-order serial cross correlations are insignificant, with the exceptions of two third-order U.K. and one Emu serial cross correlations. Partial autocorrelations and serial cross correlations at lags greater than three are not significant in these data.

TABLE 1. International Stock Market Returns

Index	Emu	W-ex-Emu	Japan	W-ex-JP	U.K.	W-ex-U.K.	U.S.	W-ex-U.S.
Observations	1304	1304	1702	1702	1745	1745	1740	1740
Daily mean return	–0.0001	–0.0001	–0.0004	0.0002	0.0000	0.0001	0.0002	–0.0001
Daily standard deviation	0.0154	0.0128	0.0134	0.0136	0.0121	0.0116	0.0128	0.0101
Skewness	–0.1608†	–0.0824	0.0726	–0.2831‡	–0.2290‡	–0.1334†	–0.1009	–0.1543‡
Excess kurtosis	1.4350‡	1.1178‡	1.7633‡	2.3165‡	2.0602‡	1.8277‡	2.4434‡	1.6892‡
Unconditional correlation	0.6693‡		0.1743‡		0.6319‡		0.4229‡	
First-order autocorrelation	0.0473†	0.1151‡	0.0309	0.1134‡	0.0227	0.1389‡	0.0023	0.1245‡
Second-order partial autocorrelation	–0.0407	–0.0341	–0.0748‡	–0.2930‡	–0.0826‡	–0.0487†	–0.0318	–0.0844‡
Third-order partial autocorrelation	–0.0640†	–0.0246	0.0028	–0.0565‡	–0.0741‡	–0.0454	–0.0362	0.0180

(*Continued*)

TABLE 1. (Continued)

Index	Emu	W-ex-Emu	Japan	W-ex-JP	U.K.	W-ex-U.K.	U.S.	W-ex-U.S.
Serial cross correlation								
First-order ($r_{x,t}$, $r_{y,t-1}$)	$0.2062^{‡}$		$0.3103^{‡}$		$0.1821^{‡}$		–0.0171	
First-order ($r_{y,t}$, $r_{x,t-1}$)		$0.0661^{‡}$		$-0.0665^{‡}$		$0.0663^{‡}$		$0.4006^{‡}$
Second-order ($r_{x,t}$, $r_{y,t-1}$)	–0.0018		–0.0189		–0.0274		–0.0147	
Second-order ($r_{y,t}$, $r_{x,t-1}$)		–0.0065		–0.0239		–0.0383		0.0085
Third-order ($r_{x,t}$, $r_{y,t-1}$)	$-0.0515^{†}$		–0.0173		$-0.0636^{‡}$		0.0177	
Third-order ($r_{y,t}$, $r_{x,t-1}$)		–0.0325		–0.0079		$-0.0474^{†}$		–0.0096

Note: Statistics based on continuously compounded, local currency daily returns from Morgan Stanley Capital International Perspectives (MSCI) markets (x = domestic and y = world-ex-domestic) over the period 02/01/1996 through 12/31/2002. The symbols † and ‡ indicate significance at 5% and 1% levels, respectively.

III. The Model

A bivariate VARMA(u,v)-EGARCH(p,q) model with a constant conditional correlation and normally distributed errors is adopted using conditional mean and volatility terms of up to three lags. This class of models is a tractable and parsimonious way to produce unconditional return distributions that fit the characteristics of observed returns to international stock indices, including significant autocorrelations and serial cross correlations at higher-order lags, time-varying means and volatilities, and asymmetric conditional volatility with relatively high comovements in the lower tails of return. The assumptions of a constant conditional correlation and normally distributed errors are popular because they are conceptually simple and computationally convenient. VARMA-in-mean terms describe the linear relation of index returns to recent returns and volatilities in that and another index.

Nelson's (1991) EGARCH model is a popular choice for modeling volatility asymmetry in univariate returns in which volatility tends to increase in response to bad news (Black [1976]; Christie [1982]; Cheung and Ng [1992]).[1] Comparisons have favored EGARCH over competing models for stock index returns in the U.S. (Pagan and Schwert [1990]; Kim and Kon [1994]; and Chen and Kuan [2002]), and Japan (Engle and Ng [1993]), emerging markets (Chong, Ahmad amd Abdullah [1999]), and small stocks (Cao and Tsay [1992]). EGARCH also has had success in modeling the implied option volatilities of stock indices (Day and Lewis [1992]).

Bivariate EGARCH has been successful in capturing interactions between an international stock index and another stock index (Koutmos, Negakis and Theodossiou [1993]; Koutmos [1996]; Booth, Martikainen and Tse [1997]; Christofi and Pericli [1999]; and Niarchos, et al. [1999]), exchange rates (Koutmos [2000]), interest rates [Lobo (2000]), and financial market liberalizations (Kassimatis [2002]); and between interest rate futures prices (Cheung and Fung [1997]; Tse and Booth [1996]; and Tse [1998]) and volumes (Jacobs and Onochie [1998]).

1. Alternatives to EGARCH for modeling asymmetric conditional volatility include (Glosten, Jaganathan and Runkle [1993]; Rabemananjara and Zakoian [1993]; and Hentschel [1995]), contemporaneous asymmetry models (Babsiri and Zakoian [2001]), stochastic volatility models (Wu [2001]), and regime-switching models (Hamilton [1989]; Fornari and Mele [1997]; and Ang and Bekaert [2002]). The EGARCH model has itself been extended in a number of ways, such as fractionally integrated EGARCH (Bollerslev and Mikkelsen [1996] and Baillie, Cecen and Han [2000]) and switching EGARCH (Daouk and Guo [2002]).

A. A Univariate MA(1)-*EGARCH*(1,1) *Baseline*

Estimation results based on bivariate models indicate that first-order VMA(1) or VAR(1) terms are sufficient for modeling the conditional means, but that second-order terms can be beneficial in the conditional volatilities. Moreover, ARCH-in-mean terms are not significant in the univariate series. Consequently, MA(1)-EGARCH(1,1) and MA(1)-EGARCH(2,2) models are estimated for each index as baselines for evaluating the bivariate models:

$$\begin{aligned} r_t &= a_0 + m_1\varepsilon_{t-1} + \varepsilon_t \\ \ln h_t &= \omega_0 + \omega_1 \ln h_{t-1} + \omega_2 \ln h_{t-2} + \lambda_1 g_{t-1} + \lambda_2 g_{t-2} \end{aligned} \tag{1}$$

where the $g_t = \gamma z_t + |z_t| - E|z_t|$ term captures the asymmetric effects of positive and negative shocks on conditional volatility. Innovations ε_t are assumed to be normally distributed, such that $\varepsilon_t \sim N(0,h_t)$ and $z_t = \varepsilon_t/\sqrt{h_t} \sim N(0,1)$.

Parameters are jointly estimated by maximum likelihood using the *BFGS* method. Maximizing a Gaussian log-likelihood function under nonnormality yields consistent estimators called quasi-maximum likelihood (*QMLE*) estimators even if the residuals are not normal (White [1982]). For testing *QMLE* estimators $\hat{\Psi}_t$, the variance-covariance matrix must be adjusted as:

$$Var\left(\hat{\Psi}_t\right) = \frac{1}{T}\left(\hat{C}_t^{-1}\hat{D}_t\hat{C}_t^{-1}\right), \tag{2}$$

where $\hat{C}_t = -(1/T)\sum_{t=1}^{T} Z_t(\hat{\Psi}_t)$, $\hat{D}_t$, $= -(1/T)\sum_{t=1}^{T} \Delta_t(\hat{\Psi}_t)'\Delta_t(\hat{\Psi}_t)$, $\Delta_t(\bullet)$ is the outer product gradient vector and $Z_t(\bullet)$ is the Hessian matrix of the log-likelihood function at time *t*, and *T* is the number of observations in the sample.

B. The Bivariate VARMA(u,v)-EGARCH(p,q) Models

Several versions of a bivariate EGARCH(*p*,*q*)-*M* with VARMA(*u*,*v*)-in -the-mean model for a domestic market *x* and a world-ex-domestic market *y* are considered,

$$\begin{aligned} &R_t = A_0 + \textstyle\sum_u A_u R_{t-u} + \sum_v M_v E_{t-v} + \Theta H_t + E_t \\ &\text{and } \ln H_t = \Omega_0 + \textstyle\sum_p \Omega_p \ln H_{t-p} + \sum_q \Lambda_q G_{t-q} \end{aligned} \tag{3}$$

for u, v, p and q up to three lags, domestic (x) and world-ex-domestic (y) returns $R_t = [r_{x,t}\ r_{y,t}]'$, innovations $E_t = [\varepsilon_{x,t}\ \varepsilon_{y,t}]'$ such that $\varepsilon_t \sim N(0,h_t)$ for each index, autoregressive conditional log volatility vector $(\ln H_t) = [\ln h_{x,t}\ \ln h_{y,t}]'$, moving average volatility vector $G_{t-1} = [g_{x,t-1}\ g_{y,t-1}]'$ such that $g_t = \gamma z_t + |z_t| - E[|z_t|]$ for $z_t = \varepsilon_t/\sqrt{h_t} \sim N(0,1)$ for each index, and ARCH-in-mean effects ΘH_t. The remaining terms are parameter matrices of the appropriate order. Following Bollerslev (1990), we assume conditional covariance is given by:

$$h_{xy,t} = \rho_{xy}\left(\sqrt{h_{x,t}}\right)\left(\sqrt{h_{y,t}}\right), \tag{4}$$

where ρ_{xy} is the constant conditional correlation between r_x and r_y.

C. Diagnostics

Ljung-Box (1978) Q statistics assess the goodness-of-fit of alternative VARMA(u,v) conditional mean specifications. The Ljung-Box Q statistic is defined by:

$$Q_L = T\left(T+2\right)\sum_{k=1}^{L}\frac{\rho_{xy,k}^2}{T-k}, \tag{5}$$

where $\rho_{xy,k}^2$ are squared sample auto- or serial cross correlations at lags from $k = 1$ to L. For a bivariate series, the Q_L statistic is asymptotically chi-square distributed with $2^2\,[L-(u+v)]$ degrees of freedom under the null hypothesis that a particular model is well specified.

Hosking (1980) extends the Q-statistic to multivariate models. In the case of a bivariate model with a maximum lag L, the multivariate portmanteau statistic is defined by:

$$P_L = T\left(T+2\right)\sum_{k=1}^{L}\frac{1}{\left(T-k\right)}Trace\left[\hat{C}_{0k}\hat{C}_{00}^{-1}\hat{C}_{00}^{'}\hat{C}_{0k}^{-1}\right] \tag{6}$$

where

$$\hat{C}_{0k} = T^{-1}\sum_{t=k+1}^{T}\left(\hat{E}_t\hat{E}_{t-k}^{'}\right).$$

For a bivariate series, Hosking's portmanteau statistic is asymptotically chi-square distributed with degrees of freedom $2^2\,[L-(u+v)]$ under the null hypothesis that the residuals are white noise. A rejection indicates that at least one of the two bivariate series is not white noise. We chose $L = 20$ after investigating various lags for the P and Q statistics.

Engle and Ng's (1993) joint bias test statistic is used to detect misspecifications related to asymmetries in the conditional volatilities. This statistic examines whether squared normalized residuals can be predicted by observed variables that are not included in the model:

$$\hat{\varepsilon}_t^2 = \varphi_0 + \varphi_1 w_{t-1}^- + \varphi_2 w_{t-1}^- \hat{\varepsilon}_{t-1} + \varphi_3 \left(1 - w_{t-1}^-\right) \hat{\varepsilon}_{t-1} + e_t \quad (7)$$

where w_{t-1}^- is a dummy variable that takes the value 1 when the residual $\hat{\varepsilon}_{t-1}$ is negative and 0 when positive. This joint bias test combines Engle and Ng's sign bias ($\varphi_1 w_{t-1}^-$), negative size bias ($\varphi_2 w_{t-1}^- \hat{\varepsilon}_{t-1}$), and positive size bias ($\varphi_3 (1 - w_{t-1}^-) \hat{\varepsilon}_{t-1}$) tests into a single nonparametric statistic. The null hypothesis H_0: $\varphi_0 = \varphi_1 = \varphi_2 = \varphi_3 = 0$ is evaluated with the test statistic TR^2 from this regression, which is asymptotically chi-square distributed with three degrees of freedom. If any of the φ_i are significant based on a one-tailed test, then equation 3 is not fully predicting the effect of the shock at time $t - 1$ on the conditional variance at time t.

Engle's (1982) LM_L statistic tests for ARCH(L) disturbances in the residuals. LM_L statistics are calculated by regressing squared standardized residuals on a constant and L lagged values of the squared residuals. The LM_L test statistic is calculated from the adjusted R^2 of this regression, $(T - L) R^2$, and is asymptotically chi-square distributed with L degrees of freedom. A lag of 4 is chosen for the LM statistic because the models have at most third-order terms. If LM_4 is significant based on a one-tailed test, then the model is not fully predicting the effects of shocks at times $t - L$ through $t - 1$ on the conditional variance at time t.

For the final model, some additional tests are applied to the conditional volatility specification. LM_L statistics at lags of $L = 2$, 3, and 4 test the various EGARCH specifications. Q_{20}^2 statistics of the conditional mean specification based on 20*th*-order autocorrelation in the squared standardized residuals test for volatility clustering in the final model. Q_L^2 statistics are simply Q_L statistics applied to squared standardized residuals and are asymptotically chi-square distributed with $2^2 [L - (u + v)]$ degrees of freedom.

A robust Wald (1943) test is conducted to see if the VARMA and EGARCH coefficients of order higher than 1 are jointly significant. To

TABLE 2. Estimated Coefficients and Diagnostic Statistics for the Univariate MA(1)-EGARCH(1,1) Models

Index	Emu	W-ex-Emu	Japan	W-ex-JP	U.K.	W-ex-U.K.	U.S.	W-ex-U.S.
Parameter estimates								
$a_{.,0}$	–0.000	–0.000	–0.001†	0.000	–0.000	–0.000	–0.000	–0.000
$m_{x.,1}$	0.073‡	0.131‡	0.060†	0.118‡	–0.052†	0.172‡	0.049‡	0.153‡
$\omega_{.,0}$	–0.239‡	–0.229†	–0.242‡	–0.183‡	–0.154‡	–0.154	–0.387‡	–0.188‡
$\omega_{x.,1}$	0.972‡	0.974‡	0.972‡	0.979‡	0.983‡	0.983‡	0.956‡	0.980‡
$\lambda_{x.,1}$	0.151‡	0.057‡	0.175‡	0.105‡	0.126‡	0.114‡	0.111‡	0.145‡
$\gamma_{.}$	–0.519‡	–1.510‡	–0.386‡	–0.815‡	–0.689‡	–0.551‡	–1.408‡	–0.622‡
Diagnostics								
Q_{20}	23.810†	13.650	20.237	22.575	17.536	22.823	17.922	17.915
Q^2_{20}	32.860‡	11.895	37.247‡	13.858	15.251	8.576	8.311	11.070
LM_L	6.145†	1.145	0.796	3.886	0.100	3.023	1.268	0.041
Joint bias test	18.004‡	2.887	4.326	4.443	1.094	3.039	6.527	5.731
Kolmogorov test	0.021	0.030‡	0.025‡	0.030‡	0.024†	0.029‡	0.026‡	0.034‡

Note: The conditional volatility specification is tested with Engle's (1982) *LM* test statistic and Engle and Ng's (1993) joint bias test statistic, which are asymptotically chi-square distributed with four and three degrees of freedom, respectively. The conditional mean specification is tested with a Ljung-Box Q_{20} statistic, which is asymptotically chi-square distributed with 2^2 (20 – 1) = 76 degrees of freedom. Wald statistics test the null hypothesis that the second-order EGARCH coefficients are all zero. Normality in the residuals is tested with a Kolmogorov statistic. Symbols † and ‡ indicate significance at 5% and 1% levels, respectively.

test the quasi-maximum likelihood parameter estimates $\hat{\theta}$ against restrictions θ_0 , note that $T^{1/2}(\theta = \theta_0)$ is asymptotically normally distributed under the null hypothesis. Squaring this and dividing by the variance of the estimate $\hat{\theta}$ yields a robust Wald statistic,

$$W = \left(\hat{\theta} - \theta_0\right)^2 / \operatorname{var}\left(\hat{\theta}\right) \tag{8}$$

that is asymptotically chi-square distributed with degrees of freedom equal to the number of restrictions being tested. A significant Wald statistic implies that higher-order coefficients are jointly significant, and that omitting them is likely to cause biased estimation.

Finally, bivariate normality in the residuals from the final models are tested with a Kolmogorov test on the univariate residuals and Mardia's skewness and kurtosis tests on the bivariate residuals. Residuals should be able to pass these normality tests if the VARMA-EGARCH model with normally distributed errors is well specified.

IV. Estimation Results

A. A Univariate Baseline for the Higher-order Terms

An MA(1)-EGARCH(1,1) model is first estimated as a baseline for evaluating the bivariate series. The choice between an AR(1) and an MA(1) conditional mean specification was not critical, as each was able to account for the conditional means. Higher-order conditional mean terms were not significant and didn't improve model performance.

Table 2 reports parameter estimates and diagnostic statistics for the MA(1)-EGARCH(1,1) model. All of the parameter estimates are significant, with the exception of a few constant terms. The persistence parameters $\omega_{x,1}$ range from 0.956 to 0.983, and the news impact parameters $\lambda_{x,1}$ range from 0.057 to 0.175. EGARCH is preferred to GARCH, as the EGARCH asymmetry term γ is negative and significant in each series. Although each of the series except the Emu has non-normal residuals, the remaining diagnostic tests reveal very few other problems. The Emu series is the exception, with poorly behaved but relatively normal residuals.

Table 3 reports estimates and diagnostic statistics for each of the

TABLE 3. Estimated Coefficients and Diagnostic Statistics for the Univariate MA(1)-EGARCH(2,2) Models

Index	Emu	W-ex-Emu	Japan	W-ex-JP	U.K.	W-ex-U.K.	U.S.	W-ex-U.S.
Parameter estimates								
$a_{.,0}$	–0.000	–0.000	–0.001†	0.000	–0.000	–0.001	–0.000	–0.000
$m_{x.,1}$	0.072‡	0.130‡	0.054‡	0.118‡	–0.052†	0.172‡	0.049‡	0.155
$\omega_{.,0}$	–0.278‡	–0.244‡	–0.282‡	–0.204‡	–0.153‡	–0.175	–0.369‡	–0.178‡
$\omega_{x.,1}$	0.847‡	0.820‡	0.927‡	0.908‡	0.991‡	0.856‡	0.959‡	0.980‡
$\lambda_{x.,1}$	0.121†	0.074†	0.087	0.074‡	0.122‡	0.107†	0.116‡	0.174†
$\gamma_{.}$	–0.482‡	–1.588‡	–0.367‡	–0.779‡	–0.688‡	–0.545‡	–1.437‡	–0.650†
$\omega_{x.,2}$	0.120	0.152‡	0.040	0.069‡	–0.008‡	0.124‡	–0.001	0.001
$\lambda_{x.,2}$	0.058	–0.013	0.111†	0.044	0.004	0.022	–0.009	–0.037

(*Continued*)

TABLE 3. (Continued)

Index	Emu	W-ex-Emu	Japan	W-ex-JP	U.K.	W-ex-U.K.	U.S.	W-ex-U.S.
Diagnostics								
Q_{20}	23.062	13.639	21.316	22.906	17.494	22.837	17.866	18.187
Q^2_{20}	28.789†	12.699	30.382‡	11.931	15.201	8.320	8.698	11.683
LM_L	4.080	1.807	1.343	2.251	0.113	2.780	1.648	0.286
Joint bias test	18.283‡	1.963	1.874	4.168	1.132	3.321	6.407	4.587
Kolmogorov test	0.022	0.028†	0.024†	0.032‡	0.024†	0.030‡	0.027‡	0.032‡
Wald test	2.090	659.779‡	5.335	472.581‡	9.078†	583.378‡	0.155	0.098

Note: The conditional volatility specification is tested with Engle's (1982) LM test statistic and Engle and Ng's (1993) joint bias test statistic, which are asymptotically chi-square distributed with four and three degrees of freedom, respectively. The conditional mean specification is tested with a Ljung-Box Q_{20} statistic, which is asymptotically chi-square distributed with $2^2 (20 - 1) = 76$ degrees of freedom. Wald statistics test the null hypothesis that the second-order EGARCH coefficients are all zero. Normality in the residuals is tested with a Kolmogorov statistic. Symbols † and ‡ indicate significance at 5% and 1% levels, respectively.

univariate series using an MA(1)-EGARCH(2,2) model. Adding second-order conditional volatility terms yields four significant persistence parameters $\omega_{x,2}$ and one significant news impact parameter $\lambda_{x,2}$. The four series that have significant $\omega_{x,2}$ terms also have significant robust Wald statistics. The residual behaviors of the second-order models are similar to those from the first-order models. As with the second-order model, the Emu is the only series that does not pass the diagnostic tests.

B. Higher-order Terms in the Bivariate Series

The partial autocorrelations and serial cross correlations in table 1 suggest that coefficients on the lagged AR_{t-1}. and ME_{t-1} terms are likely to be important, and that the importance of these terms may differ across market pairs. An exhaustive search through the class of VARMA(*u*,*v*)-EGARCH(*p*,*q*) models for arbitrary lags 0 through L would include $(L + 1)^4$ distinct model specifications. For a maximum lag of $L = 3$, this would include $(3 + 1)^4 = 256$ model estimations.

To reduce the search space, assume that the conditional mean and conditional volatility structures are independent beyond one lag and insignificant beyond three lags. This allows a separate search through the set of VARMA(*u*,*v*)-EGARCH(1,1) and VARMA(1,1)- EGARCH(*p*,*q*) models using a maximum lag of 3. This assumption reduces the search space to $2(3 + 1)^2 = 32$ model specifications. Diagnostic statistics for VARMA(0,0) and EGARCH(0,0) models are reported as a baseline. Diagnostic statistics reject these unconditional models at a 1 percent significance level in each case.

A search is conducted for the most parsimonious model with insignificant probability values (*p*-values) on the diagnostic statistics, imposing this criterion separately for the EGARCH and VARMA searches. In particular, a search is conducted for the most parsimonious VARMA(1,1)-EGARCH(*p*,*q*) model according to the *p*-values of the LM_4 and joint bias statistics across the various EGARCH specifications. Another search is then conducted for the most parsimonious VARMA(*u*,v)-EGARCH(1,1) model that minimizes the maximum misspecification in the *p*-values of the Q_{20} and P_{20} statistics.

Wald statistics are calculated relative to the first-order model in each variable. Models with lags greater than (1,1) are compared to the (1,1) model. Models with only a *p* or *u* term are compared to a (1,0) baseline. Models with only a *q* or *v* term are compared to (0,1). The Wald statistic thus provides a test of whether higher-order terms are jointly significant relative to the comparable first-order (1,0), (0,1) or (1,1) model. Note that a significant Wald statistic in a second-order model will generally result in a significant Wald statistic in similar third-order models because all higher-order terms are compared to a first-order base.

Table 4a reports LM_4, joint bias and Wald tests for the various VARMA(1,1)-EGARCH(p,q) candidate models and market pairs. Table 4b reports p-values for these diagnostic statistics. Some general conclusions are apparent from the tables. First, diagnostic statistics tend toward insignificance at higher lags in each series. Also, at least one lagged volatility term is necessary to introduce persistence into estimates of conditional volatility, as specifications from (0,0) to (0,3) are poorly behaved across all four series. EGARCH(1,0), (2,0), or (3,0) are not considered, as these models reduce to a constant-variance model.

EGARCH(1,1) works well for the U.K. and U.S. series in tables 4a and 4b, rendering the diagnostic tests insignificant in these series. Wald tests for joint significance in the higher-order terms for the U.K. and U.S. series are significant, but the other diagnostic tests show that EGARCH(1,1) is sufficient to accommodate the observed volatility persistence in these series.[2]

Higher-order terms are necessary for Japan, where the LM_4 statistics indicate ARCH(4) disturbances in the residuals of the EGARCH(1,1) model. Lags of at least (1,3) or (2,2) are necessary to remove significance in the LM_4 statistics. EGARCH(2,2) is adopted as the parsimonious model for Japan, although (1,3) works as well. The significant Wald statistics for Japan confirm that the higher-order terms are jointly significant.

Tables 4a and 4b also suggest that a higher-order EGARCH term can improve the fit of the Emu series. In particular, lags of (1,2) or (2,1) or higher are necessary to remove significance in the LM_4 statistics. Whether the second-order lag is on previous volatility or innovation does not seem to matter. Note that higher-order lags cannot remove the significance in the joint bias statistics for the Emu, although EGARCH(0,q) models with $q \geq 1$ do improve the behavior of the squared normalized residuals. Choice of the best model for this series is difficult, as the Wald statistic is significant for the EGARCH(2,1) specification, but not for the EGARCH(1,2) specification. The next section assesses whether EGARCH(2,1) can improve on EGARCH(1,1)

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2. Diagnostic p-values that exceed 0.95 for the U.S. EGARCH(0,2) and EGARCH(2,2) models merely indicate that residual autocorrelations are smaller than normal, and hence benign.

TABLE 4A. Diagnostic Statistics for the EGARCH(*p*,*q*) Specifications

Index	Test (*p*,*q*) -	(0,0)	(0,1)	(0,2)	(0,3)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
Emu	LM_4	173.2‡	172.4‡	106.5‡	20.6‡	11.0†	**4.6**	2.4	**7.1**	6.9	2.4	10.6†	9.9†	2.6
	Joint bias	25.4‡	2.6	3.5	5.4	13.7‡	**11.6**‡	10.4†	**14.2**‡	11.6‡	8.3†	14.5‡	12.0‡	11.6‡
W-ex-Emu	LM_4	57.2‡	45.4‡	14.1‡	7.3	2.9	**4.8**	3.1	**5.0**	4.4	3.5	4.8	3.7	3.5
	Joint bias	22.1‡	2.4	2.0	1.1	4.5	**1.8**	1.6	**4.0**	1.5	1.3	1.4	1.7	2.4
	Wald	n/a	n/a	22.5‡	44.3‡	n/a	**5.12**	10.7	**1.6e3**‡	1.5e5‡	3.3e5‡	9.0e5‡	1.9e6‡	4.2e5‡
Japan	LM_4	80.4‡	96.6‡	52.9‡	5.0	31.6‡	20.0‡	**4.0**	18.5‡	**4.2**	6.4	15.0‡	5.6	3.0
	Joint bias	20.6‡	3.3	0.5	1.1	0.6	0.9	**1.0**	6.2	**1.7**	1.1	4.2	1.0	1.3
W-ex-Japan	LM_4	110.5‡	59.2‡	41.0‡	15.9‡	4.3	3.9	**3.4**	5.4	**4.9**	4.0	4.9	2.7	2.9
	Joint bias	24.5‡	5.3	6.2	5.5	3.8	4.2	**4.7**	4.6	**5.1**	5.2	6.3	4.9	4.8
	Wald	n/a	n/a	21.7‡	116.0‡	n/a	16.6‡	**47.8**‡	1.3e5‡	**643.1**‡	1.2e3‡	8.7e6‡	8.1e5‡	4.5e6‡
U.K.	LM_4	255.1‡	160.2‡	151.5‡	68.3‡	**1.6**	2.4	1.9	3.6	2.1	2.6	3.7	2.2	1.2
	Joint bias	53.6‡	2.6	1.6	2.2	**3.2**	3.6	3.3	2.2	3.0	3.2	2.9	3.5	3.8
W-ex-U.K	LM_4	435.2‡	66.9‡	28.8‡	19.2‡	**1.8**	2.4	2.0	3.3	1.6	1.9	4.8	2.1	1.5
	Joint bias	49.4‡	1.3	3.4	2.1	**3.2**	3.4	2.6	4.2	2.9	2.9	3.5	2.6	2.5
	Wald	n/a	n/a	72.8‡	105.6‡	**n/a**	8.8	10.6	1.7e9‡	1.5e4‡	1.7e4‡	9.0e7‡	3.5e6‡	2.4e6‡
U.S.	LM_4	110.5‡	64.8‡	47.8‡	16.1‡	**1.8**	1.7	4.5	1.9	0.5	4.5	3.0	1.4	4.3
	Joint bias	67.9‡	1.7	0.2	1.0	**6.1**	4.0	6.6	6.3	5.6	7.1	7.1	7.2	6.9
W-ex-U.S.	LM_4	179.9‡	112.8‡	94.6‡	56.3‡	**8.5**	11.6†	8.7	9.9†	7.7	9.3	7.1	9.5	5.1
	Joint bias	71.8‡	3.3	6.0	6.9	**3.7**	6.9	6.4	3.2	7.3	5.4	5.9	5.9	7.4
	Wald	n/a	n/a	154.0‡	80.9‡	**n/a**	22.8‡	18.8†	2.2e3‡	5.1e3‡	3.6e5‡	1.4e7‡	3.9e6‡	5.5e5‡

Note: Each conditional volatility specification is tested with Engle's (1982) LM test statistic and Engle and Ng's (1993) joint bias test statistic. LM_4 and joint bias test statistics are asymptotically chi-square distributed with four and three degrees of freedom, respectively. Wald statistics test the null hypothesis that the EGARCH coefficients of order higher than 1 are all zero. Exponential notation "eN" is equivalent to "x10^N". Symbols † and ‡ indicate significance at 5% and 1% levels, respectively. Parsimonious models are indicated in bold.

TABLE 4B. P-Values of the Diagnostic Statistics for EGARCH(*p*,*q*) Specifications

Index	Test	(*p*,*q*) - (0,0)	(0,1)	(0,2)	(0,3)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
Emu	LM_4	0.000	0.000	0.000	0.000	0.026	**0.337**	0.659	**0.129**	0.143	0.656	0.031	0.043	0.620
	Joint bias	0.000	0.452	0.320	0.147	0.003	**0.009**	0.016	**0.003**	0.009	0.040	0.002	0.007	0.009
W-ex-Emu	LM_4	0.000	0.000	0.007	0.121	0.577	**0.313**	0.539	**0.288**	0.356	0.482	0.308	0.449	0.477
	Joint bias	0.000	0.481	0.568	0.769	0.217	**0.613**	0.664	**0.262**	0.688	0.737	0.709	0.623	0.497
	Wald	n/a	n/a	0.000	0.000	n/a	**0.275**	0.221	**0.000**	0.000	0.000	0.000	0.000	0.000
Japan	LM_4	0.000	0.000	0.000	0.288	0.000	0.001	**0.408**	0.000	**0.376**	0.169	0.005	0.233	0.563
	Joint bias	0.000	0.345	0.912	0.788	0.905	0.836	**0.791**	0.103	**0.642**	0.767	0.236	0.796	0.742
W-ex-Japan	LM_4	0.000	0.000	0.000	0.003	0.370	0.416	**0.489**	0.246	**0.296**	0.401	0.297	0.608	0.568
	Joint bias	0.000	0.151	0.102	0.138	0.282	0.236	**0.198**	0.204	**0.168**	0.160	0.098	0.177	0.188
	Wald	n/a	n/a	0.000	0.000	n/a	0.002	**0.000**	0.000	**0.000**	0.000	0.000	0.000	0.000
U.K.	LM_4	0.000	0.000	0.000	0.000	**0.808**	0.640	0.765	0.415	0.715	0.631	0.554	0.627	0.935
	Joint bias	0.000	0.452	0.662	0.542	**0.399**	0.280	0.345	0.734	0.372	0.388	0.614	0.313	0.290
W-ex-U.K.	LM_4	0.000	0.000	0.000	0.000	**0.769**	0.643	0.744	0.524	0.800	0.754	0.416	0.725	0.849
	Joint bias	0.000	0.737	0.338	0.552	**0.374**	0.378	0.467	0.137	0.407	0.389	0.204	0.448	0.484
	Wald	n/a	n/a	0.000	0.000	**n/a**	0.066	0.223	0.000	0.000	0.000	0.000	0.000	0.000
U.S.	LM_4	0.000	0.000	0.000	0.003	**0.783**	0.788	0.343	0.768	0.977	0.324	0.563	0.849	0.366
	Joint bias	0.000	0.646	0.975	0.793	**0.105**	0.264	0.084	0.098	0.122	0.070	0.068	0.066	0.077
W-ex-U.S.	LM_4	0.000	0.000	0.000	0.000	**0.075**	0.020	0.069	0.038	0.102	0.069	0.129	0.051	0.275
	Joint bias	0.000	0.342	0.113	0.077	**0.292**	0.074	0.095	0.360	0.094	0.136	0.116	0.115	0.060
	Wald	n/a	n/a	0.000	0.000	**n/a**	0.000	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Note: Each conditional volatility specification is tested with Engle's (1982) *LM* test statistic and Engle and Ng's (1993) joint bias test statistic. LM_4 and joint bias test statistics are asymptotically chi-square distributed with four and three degrees of freedom, respectively. Wald statistics test the null hypothesis that the EGARCH coefficients of order higher than 1 are all zero. Parsimonious models are indicated in bold.

TABLE 5A. Diagnostic Statistics for the VARMA(u,v) Specifications

Market index	(u,v)	(0,0)	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)	(3,2)	(3,3)
Emu	Q_{20}	31.3†	**27.4**†	25.8†	22.0†	**29.5**†	26.4†	22.7†	22.2†	26.8†	22.2†	22.9†	25.8‡	21.7†	22.1†	21.3‡	19.9‡
W-ex-Emu	Q_{20}	35.5‡	**14.9**	15.3	13.1	**15.7**	16.3	16.3	16.1	15.2	16.1	15.8	16.8†	13.6	16.6	11.2	13.9†
	P_{20}	146.3‡	**71.0**	64.8	59.1	**78.1**	65.1	56.2	58.0	68.4	55.5	58.5	60.3	59.2	55.7	50.8	57.5
	Wald	n/a	**n/a**	25.5‡	82.3‡	**n/a**	n/a	23.7‡	188.9‡	17.0‡	1.2e3‡	8.4e3‡	1.3e5‡	39.9‡	35.6‡	382.8‡	4.6e4‡
Japan	Q_{20}	32.6†	**22.7**	21.4	20.4	**24.2**	20.0	15.4	13.5	22.1	19.2	5.9	6.7	21.8†	12.2	6.7	5.1
W-ex-Japan	Q_{20}	41.5‡	**22.1**	22.9	18.9	**22.1**	25.8†	21.2†	16.9	21.5	16.9	19.4†	18.3†	18.5	16.0	18.3†	17.8‡
	P_{20}	200.7‡	**79.0**	78.0	71.5	**80.5**	85.8	69.2	58.2	78.7	67.3	51.2	49.1	71.2	59.0	48.5	49.0
	Wald	n/a	**n/a**	6.3	16.9†	**n/a**	n/a	83.4‡	30.3‡	11.6†	29.3‡	2.4e5‡	1.3e4‡	31.5‡	2.7e3‡	6.9e3‡	3.7e3‡
U.K.	Q_{20}	25.5	**17.3**	14.8	12.0	**19.9**	22.1	19.5	13.5	15.9	13.8	10.4	14.2	6.5	15.9	12.5	16.8†
W-ex-U.K.	Q_{20}	66.0‡	**25.6**	24.6†	18.3	**25.1**	24.3†	16.4	15.8	24.2†	23.8†	25.8‡	24.1‡	16.8	20.5†	20.3‡	17.6‡
	P_{20}	145.6‡	**75.4**	71.9	64.0	**76.4**	78.0	69.0	62.3	72.3	65.5	70.4	67.2	44.7	69.4	65.0	73.0†
	Wald	n/a	**n/a**	12.9†	27.0‡	**n/a**	n/a	1.5e3‡	1.3e3‡	8.7	45.6‡	2.8e6‡	582.2‡	10.0	3.7e3‡	1.2e5‡	6.3e6‡
U.S.	Q_{20}	20.4	**17.7**	18.1	16.3	**18.3**	17.3	15.0	16.8	19.4	15.3	15.5	11.3	17.8	14.8	12.2	14.2‡
W-ex-U.S.	Q_{20}	52.4‡	**18.5**	17.4	17.0	**21.3**	17.1	21.4†	13.7	17.7	21.7†	18.7†	19.6†	15.2	17.0	19.3†	16.2‡
	P_{20}	341.5‡	**77.0**	67.5	59.5	**79.1**	71.4	64.1	53.8	73.9	61.2	60.1	56.1	62.8	64.8	55.0	58.5‡
	Wald	n/a	**n/a**	7.5	17.9†	**n/a**	n/a	2.8e3‡	738.0‡	10.3†	207.8‡	2.0e3‡	7.1e3‡	27.1‡	796.0‡	2.4e4‡	3.1e13‡

Note: The conditional mean specifications are tested with Ljung-Box Q_{20} statistics for the (domestic and world-ex-domestic) residuals and Hosking's multivariate portmanteau P_{20} statistics for the first 20 lags of the standardized residuals from each model. Both statistics are asymptotically chi-square distributed with $2^2(20 - u + v)$ degrees of freedom. Wald statistics test the null hypothesis that the VARMA coefficients of order higher than 1 are all zero. Exponential notation "eN" is equivalent to "x10^N". Symbols † and ‡ indicate significance at 5% and 1% levels, respectively. Parsimonious models are indicated in bold.

TABLE 5B. *P*-Values of the Diagnostic Statistics for Various VARMA(*u*,*v*)-EGARCH(1,1) Specifications

Market index	(*u*,*v*)	(0,0)	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)	(3,2)	(3,3)
Emu	Q_{20}	0.026	**0.037**	0.027	0.037	**0.021**	0.023	0.030	0.035	0.020	0.035	0.011	0.001	0.041	0.014	0.006	0.003
W-ex-Emu	Q_{20}	0.008	**0.534**	0.356	0.359	**0.477**	0.298	0.176	0.187	0.363	0.187	0.106	0.032	0.330	0.083	0.191	0.030
	P_{20}	0.000	**0.512**	0.588	0.649	**0.291**	0.577	0.745	0.549	0.464	0.766	0.530	0.323	0.648	0.632	0.672	0.278
	Wald	n/a	**n/a**	0.000	0.000	**n/a**	n/a	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Japan	Q_{20}	0.019	**0.121**	0.093	0.060	**0.085**	0.130	0.222	0.196	0.077	0.084	0.823	0.571	0.039	0.272	0.573	0.534
W-ex-Japan	Q_{20}	0.001	**0.140**	0.062	0.090	**0.139**	0.028	0.048	0.076	0.088	0.154	0.035	0.019	0.101	0.099	0.019	0.007
	P_{20}	0.000	**0.268**	0.191	0.244	**0.231**	0.071	0.307	0.542	0.175	0.366	0.784	0.733	0.252	0.513	0.752	0.593
	Wald	n/a	**n/a**	0.176	0.032	**n/a**	n/a	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000
U.K.	Q_{20}	0.111	**0.366**	0.390	0.446	**0.226**	0.076	0.078	0.198	0.319	0.313	0.409	0.078	0.890	0.101	0.131	0.010
W-ex-U.K	Q_{20}	0.000	**0.060**	0.039	0.107	**0.068**	0.043	0.173	0.106	0.043	0.022	0.004	0.002	0.157	0.025	0.009	0.007
	P_{20}	0.000	**0.370**	0.349	0.477	**0.340**	0.190	0.313	0.393	0.336	0.424	0.169	0.145	0.968	0.190	0.192	0.029
	Wald	n/a	**n/a**	0.012	0.001	**n/a**	n/a	0.000	0.000	0.068	0.000	0.000	0.000	0.263	0.000	0.000	0.000
U.S.	Q_{20}	0.309	**0.343**	0.203	0.178	**0.309**	0.241	0.239	0.078	0.149	0.226	0.116	0.187	0.123	0.140	0.141	0.028
W-ex-U.S.	Q_{20}	0.000	**0.296**	0.235	0.150	**0.167**	0.251	0.045	0.188	0.219	0.041	0.045	0.012	0.229	0.075	0.013	0.013
	P_{20}	0.000	**0.323**	0.494	0.636	**0.264**	0.366	0.474	0.702	0.291	0.577	0.472	0.471	0.518	0.312	0.513	0.249
	Wald	n/a	**n/a**	0.113	0.022	**n/a**	n/a	0.000	0.000	0.036	0.000	0.000	0.000	0.001	0.000	0.000	0.00

Note: The conditional mean specifications are tested with Ljung-Box Q_{20} statistics for the (domestic) residuals and Hosking's multivariate portmanteau P_{20} statistics for the first 20 lags of the standardized residuals from each model. Each statistic is asymptotically chi-square distributed with $2^2(20 - u + v)$ degrees of freedom. Wald statistics test the null hypothesis that the VARMA coefficients of order higher than 1 are all zero. Parsimonious models are indicated in bold.

TABLE 6. Estimated Coefficients and Diagnostic Tests for the VMA(1)-EGARCH(*p*,*q*) Models

	x Emu	*y* W-ex-Emu	*x* Japan	*y* W-ex-JP	*x* U.K.	*y* W-ex-U.K.	*x* U.S.	*y* W-ex-U.S.
Parameter estimates								
$a_{.,0}$	–0.001	–0.002	–0.001†	–0.000	–0.001	–0.001	–0.001	–0.001
$m_{x.,1}$	–0.140‡	0.321‡	0.017	0.301‡	–0.088‡	0.226‡	0.034	0.051
$m_{y.,1}$	0.004	0.122‡	–0.044†	0.135‡	–0.027	0.177‡	0.319‡	–0.012
$\theta_{.}$	6.032	11.261	7.444†	2.127	6.296	5.320	7.867	8.455
$\omega_{.,0}$	–0.731‡	–0.692‡	–0.734‡	–0.173	–0.295	–0.341	–0.625‡	–0.326‡
$\omega_{x.,1}$	0.875‡	0.161‡	0.550‡	–1.892‡	0.931‡	0.037	0.917‡	0.011
$\omega_{y.,1}$	0.291‡	0.486‡	–0.177‡	0.472‡	0.078‡	0.884‡	–0.004	0.970‡
$\lambda_{x.,1}$	0.149‡	0.007	0.078	0.089‡	0.130‡	0.004	0.105‡	0.046
$\lambda_{y.,1}$	0.046	0.021	0.101†	0.060	0.054	0.111	0.031	0.139‡
$\gamma_{.}$	–0.469‡	–4.190	–0.336‡	–1.212‡	–0.686†	–0.457	–1.530‡	–0.384‡
$\omega_{x.,2}$	0.141‡	–0.259‡	0.419‡	1.839‡				
$\omega_{y.,2}$	–0.240‡	0.385‡	–0.149‡	0.480‡				
$\lambda_{x.,2}$			0.275‡	0.108†				
$\lambda_{y.,2}$			–0.053	0.048†				

(*Continued*)

TABLE 6. (Continued)

	x Emu	*y* W-ex-Emu	*x* Japan	*y* W-ex-JP	*x* U.K.	*y* W-ex-U.K.	*x* U.S.	*y* W-ex-U.S.
Diagnostics								
Q_{20}	28.864	15.952	24.361	22.028	17.283	25.632	17.673	18.488
Q^2_{20}	27.361	13.986	20.113	13.309	21.566	7.453	9.385	18.555
P_{20}	74.230		78.336		75.543		76.972	
LM_L	12.826‡	1.697‡	13.893‡	2.783	0.042	1.468	1.877	1.195
Joint bias test	14.163‡	1.599	0.782	4.440	0.986	3.285	5.976	4.590
Kolmogorov test	0.026†	0.032‡	0.019	0.030‡	0.025†	0.028‡	0.026‡	0.032‡
Mardia skewness	0.080‡		0.104‡		0.198‡		0.118‡	
Mardia kurtosis	10.078‡		9.477‡		10.622‡		9.535‡	
Wald test	2156.68‡		684.55‡		n/a		n/a	

Note: Symbols † and ‡ indicate significance at 5% and 1% levels, respectively.

for the Emu series. This model is chosen rather than EGARCH(1,2) because of its significant Wald statistic in table 4a.

Tables 5a and 5b report Q_{20}, P_{20} and *Wald* statistics for the various VARMA(u,v) conditional mean models. The main conclusion here is that VAR(1) and VMA(1) are equally capable of removing intertemporal dependencies from the residuals. Indeed, diagnostic statistics and their p-values are nearly identical across all four series for these two specifications. Some conditional mean specification is necessary, as the VARMA(0,0) specification is a poor fit for each series. A VMA(1) conditional mean specification is adopted following Burns, Engle and Mezrich (1998), although VAR(1) works as well. The final, parsimonious models have a VMA(1) conditional mean and a conditional volatility specification of EGARCH(2,1) for the Emu, EGARCH(2,2) for Japan, and EGARCH(1,1) for the United Kingdom and the United States.

V. A Parsimonious Model of Bivariate Returns to International Stock Indices

Table 6 shows parameter estimates and diagnostic statistics for the best-fitting VMA(1)-EGARCH(p,q) model for each series. With the exception of Japan, the constant coefficients $a_{x,0}$ and $a_{y,0}$ are not statistically significant, so these typically do not show a trend over the sample period. The $\theta_{.}$ terms also are not significant, so the level of return is not related to volatility except through the conditional volatility specification. This is consistent with most previous estimates of ARCH-in-mean effects.

These indices exhibit predominantly positive moving average terms in table 6. The strongest effects are in the serial cross terms $m_{xy,1}$ and $m_{yx,1}$, which are positive and significant whenever one index closes before another. For example, positive and significant $m_{xy,1}$ terms for the domestic Emu (0.319), Japan (0.301), and U.K. (0.226) indices reflect information from world-ex-domestic markets (such as the U.S.) that arrives after the close of the domestic market and is included in the next day’s domestic return. Similarly, the positive and significant cross effect $m_{yx,1}$ for the U.S. series (0.319) arises because world-ex-U.S. returns include information from the previous day’s U.S. return. There are also positive and significant moving average terms $m_{yy,1}$ for the world-ex-Emu (0.101), world-ex-Japan (0.135) and world-ex-U.K. (0.177) indices.

The negative and significant moving average terms in the Emu (–0.137) and U.K. (–0.088) appear at odds with the other moving average terms. However, the sum $m_{xx,1} + m_{xy,1}$ is positive for the Emu (0.182), Japan (0.318), the U.K. (0.138), and the U.S. (0.085), so the combined effect of serial and serial cross terms on domestic return is positive for each domestic index.

The indices exhibit strong volatility persistence. All eight $\omega_{xx,1}$ and $\omega_{yy,1}$ terms on lagged log variances are positive and significant. The $\omega_{xx,1}$ terms are close to unity for the first-order terms of the U.K. (0.931) and U.S. (0.917) indices. Similarly, the $\omega_{yy,1}$ terms are close to unity for the first-order terms of the world-ex-U.K. (0.884) and world-ex-U.S. (0.970) indices.

In the Emu, the effect is spread over two lags and often appears in the cross effects. Each of the first-order autoregressive volatility terms $w_{xx,1}$ and $w_{yy,1}$ are positive and significant, but the sums $(\omega_{xx,1} + \omega_{xx,2}) =$ 0.548 and $(\omega_{yy,1} + \omega_{yy,2}) = 0.750$ are not as close to unity as in the other series. The world-ex-Emu index also has a positive and significant second-order $\omega_{yy,2}$ term. Positive and significant cross effects $\omega_{xy,1} =$ 0.744 and $\omega_{yx,1} = 0.125$ are not a surprise, as Emu markets close after Japan and before the U.S. Second-order cross effects are not significant.

Similarly, in Japan the sums $(\omega_{xx,1} + \omega_{xx,2}) = 0.969$ and $(\omega_{yy,1} + \omega_{yy,2})$ = 0.952 are close to unity. The cross effects $\omega_{xy,1}$ and $\omega_{xy,2}$ have nearly the same absolute value but opposite sign, so the $\omega_{xy,1} = –1.892$ impact of volatility in Japan on world-ex-Japan volatility at lag one is canceled out by the cross effect $\omega_{xy,2} = –1.839$ at lag two. The impact of world-ex-Japan volatility on Japanese volatility is negative and significant at lags one ($\omega_{yx,1} = –0.177$) and two ($\omega_{yx,2} = –0.149$), although the magnitude of this cross effect is far less than the $(\omega_{xx,1} + \omega_{xx,2}) =$ 0.969 magnitude of the serial effect within the domestic Japan index.

The $\gamma_{\cdot}$ coefficients are negative and mostly significant, so the positive $\lambda_{\cdot}$ terms mean that conditional variances are larger when previous innovations are negative than when they are positive. This is a common finding in many asset prices, including international stock indices (Kroner and Ng [1998]). The $\lambda_{xx,1}$ and $\lambda_{yy,1}$ terms are positive, and generally significant for the first-order models of the U.K. (0.130), and the U.S. (0.105 and 0.139), so negative innovations in each index have a larger influence on conditional variance than positive innovations. There is mixed evidence of asymmetric volatility traveling between the indices in the first-order models of the Emu, U.K., and U.S., with a single positive and significant cross effect $\lambda_{yx,1}$ for world-ex-Emu innovations on Emu returns.

For Japan's EGARCH(2,2) model, positive and significant $\lambda_{xx,2}$ and $\lambda_{yy,2}$ terms appear at lag two but not at lag one. In contrast to the U.K. and U.S. series, three of the four cross effects ($\lambda_{xy,1}$ = 0.089, $\lambda_{yx,1}$ = 0.101, and $\lambda_{xy,2}$ = 0.108) are positive and significant in the Japan series. The other cross effect ($\lambda_{yx,2}$ = –0.053) is not significant. As in the other series, conditional variances are larger when previous innovations are negative than when they are positive. However, the relation travels both within and across indices and continues for two lags in the Japan series.

Overall, the diagnostics tests in table 6 indicate that an VMA(1)-EGARCH(1,1) model captures most of the characteristics of the U.K. and U.S. series. Diagnostic statistics for the Emu's VMA(1)-EGARCH(2,1) model and Japan's VMA(1)-EGARCH(2,2) model are slightly more problematic. All of the test statistics should be insignificant if a model is well specified. Ljung-Box Q_{20} statistics on standardized VMA(1) residuals are insignificant for each index. Each bivariate series passes Hosking's portmanteau test P_{20} of the VMA(1) conditional mean specification. Q^2_{20} tests on squared standardized VMA(1) residuals and LM_L tests for ARCH(L) disturbances reveal no problems, with the exception of the LM_3 test for Japan. The domestic Emu index is the only one that fails the joint bias test. The world-ex-domestic residuals and the bivariate residuals are unable to pass the normality tests at a one percent significance level, suggesting that an alternative error distribution might be worth exploring.

In summary, a first-order vector autoregressive or moving average process is sufficient to model conditional mean returns in these international stock indices. Although first-order conditional volatility terms are sufficient in two of the four series, second-order terms are significant in the bivariate Emu and Japan series. Robust Wald statistics on the second-order terms are significant relative to the EGARCH(1,1) baseline, and residuals are poorly behaved without the second-order terms. Finally, the univariate and the bivariate series generally have a negative and significant asymmetric volatility term in the EGARCH model, indicating a greater volatility response to negative innovations than to positive innovations.

VI. Conclusions and Suggestions for Future Research

This article documents the stochastic properties of bivariate returns to MSCI's domestic and world-ex-domestic stock index pairs for the Emu, Japan, the United Kingdom, and the United States. Bivariate returns to these series are important because they determine the diversification gains to domestic investors from international equity investments. A search is conducted for higher-order terms in the class of bivariate VARMA(u,v)-EGARCH(p,q) models with a constant conditional correlation and normally distributed errors.

Higher-order conditional volatility terms can be significant in these data. A VMA(1)-EGARCH(1,1) model provides a relatively good fit for the U.K. and U.S. series. However, higher-order EGARCH terms and robust Wald statistics are significant in the Emu and Japan. This is similar to the proportion of significant higher-order terms in the univariate series.

This study could be extended in many ways, as findings are limited by the assumptions and data. For example, it may be fruitful to investigate alternative conditional correlation structures or error distributions, as Bollerslev, Engle and Wooldridge (1988) and King, Sentana and Wadhwani (1994) find that stock index correlations vary over time with higher correlations in bear markets (Longin and Solnik [2001]; Butler and Joaquin [2002]; Bae, Karolyi and Stulz [2003]). Nonnormal error distributions might prove useful (Liesenfeld and Jung [2000]), such as the skewed generalized T (Theodossiou [1998]), stable paretian (Mittnik, Paolella and Rachev [2002]), exponential generalized beta (Wang, et. al. [2001]), or generalized error distribution (Nelson [1988]). Also, higher-order conditional volatility lags could be investigated in bivariate series that involve higher transaction costs, more price adjustment delays, or lower liquidity than the large markets examined in this study.

Although the statistical significance of higher-order conditional volatility terms is demonstrated in this study, their economic significance is not. The economic significance of a more precise model of conditional volatility is a potentially fruitful area of research and is receiving increasing attention in the literature. For example, Fleming, Kirby and Ostdiek (2001, 2003) assess whether an improved volatility model can lead to better asset allocation decisions and estimate that a first-order model of conditional volatility is worth 50 to 200 basis points per year to a risk-averse investor relative to an unconditional volatility model. The economic significance of higher-order conditional volatility terms could be assessed in a similar manner.

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